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Original Article

Vibration Analysis of Cracked Cantilever Beam for Various Crack Depths, Crack Locations and Crack Opening Size by Finite Element Method

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Abstract *- Any structure with the presence of a crack is responsive to collapse due to a change in dynamic behaviour as the crack introduces flexibility, and thus, mode shape natural frequency is reduced due to the reduction of structural stiffness value. Change in dynamic behaviour leads to resonance formation. Hence, it is very crucial to govern the natural frequency of vibration. In this vibration analysis paper, mode shapes and natural frequencies of lateral vibration are extracted for cracked, free-cracked cantilever beams for five mode shapes. Euler-Bernoulli's beam theory is considered to govern both natural frequencies and the shape of a cracked free cantilever beam. The natural frequency values of cracked free beam obtained from the Finite Element Method (FEM) and Euler-Bernoulli's Beam Theory are compared. Transverse vibration analysis is expanded to analyse the effect of crack depth, position and opening size on natural frequency. As the cracked beam's modal analysis is difficult to solve using an analytical method, software based on Finite Element Analysis (FEA) "Ansys Workbench 17.0" is taken into account for numerical analysis. The study analyses how the cracked beam's natural frequencies for different mode shapes alter due to different situations, such as variations of crack depth, location, and opening size during vibration.*

Keywords *- Cantilever beam, Modal natural frequency, Free vibration, Crack identification, FEA.*

1. Introduction

Almost all structures have defects like cracks, which may result from various reasons, such as service conditions, production processes, etc. For these reasons, high-stress levels are induced in overstressed zones, and failure may arise. Hence, cracks should be identified in a structure as soon as possible because dynamic response characteristics are changed. Dynamic response characteristics should be diagnosed for both damaged and undamaged structures like cantilever beam. Cracks in a cantilever beam are responsible for local stiffness variation when its location, depth and opening size are altered. The performance, safety, and integrity of a cantilever beam should be taken care of, which is possible by tracking the variation of dynamic response parameters. Fluctuations found in response phenomenon can be diagnosed by checking whether the crack is open, closed or breathing in nature. There are many non-destructive techniques by which all types of information about interior regions may be explored without damaging the beam. Techniques are magnetic particle testing, liquid penetration method, eddy current method, modal analysis, ultrasonic testing, etc. Among these, modal analysis is the most effective, easier and cost-effective tool for vibration analysis of cracked cantilever beam. Modal analysis means finding different modal natural frequencies of both cracked and cracked free beams. When there is a crack in a beam, it causes discontinuities. Discontinuities cause variations in structural stiffness and mode shape frequency changes. A cracked beam exists with discontinuities, which is impossible to analyse through analytical methods. FEM is the best one for solving this type of analysis computationally. In most of the literatures, it is assumed that cracks always remain open when vibrating. When there is a dynamic loading, assumption may seem to be invalid. In such situations, the crack opens and closes regularly, leading to variation in beam stiffness and the presence of dynamic behaviour, which is nonlinear. The presence of cracks severely destroys the execution of a m/c or structure. The prime cause of their failure is the material fatigue phenomenon. The main focus of several studies is developing techniques that can easily identify localized cracks in a structures like beams.

2. Literature Review

Researchers have done their research works on cracked beam vibration in many research organisations all over the world [1-9]. Dynamic behaviours of cracked cantilever beams are significantly impacted by factors leading to localized variation in stiffness. The formation of cracks is responsible for changes in natural frequencies during vibrations, amplitude of forced vibrations and dynamic stability conditions [1-9]. It's not impossible to identify the presence of cracks in structures like beam without dismantling the entire system. The only way to achieve this solution is by measuring mode shape-changing data due to the presence of defects like cracks. In an appreciable work, several crack modelling approaches were reviewed by Dimarogonas. These approaches were named local bending moment, equivalent reduced cross-section method, crack identification method, and local flexibility, which are applied to cracked beams and rotors using experimental, analytical, and numerical methods.

A differential equation with related boundary conditions of a beam having the presence of various symmetric cracks by applying Euler-Bernoulli's beam theory was deduced by Barr and Christides [10]. A crack in beam causes uncertainty in stress, which is taken into account by introducing a function. A parameter inside this function was analysed experimentally. Value of which decays exponentially from the crack location. They showed that the results of the analytical investigation procedure match very closely with experimental data. A series of comparison functions consisting of mode shapes of the corresponding cracked free beam were considered. Proposed differential Eq. is very much effective in finding out higher order mode shape(s) and modal frequency(s) of cracked beam. For validating their theoretical findings, 2D FEM was used.

Research work done by Barr and Christides [10] was enhanced by Pierre and Shen [11] by taking help to incorporate an approximate Galerkin soln of beam having two symmetric cracks which always remain open during vibration. For this analysis, a set of comparison functions were used. These functions consist of identical un-cracked beam' mode shapes. Unlike Barr and Christides [10] their proposed method is also helpful for finding out higher order frequencies as well as cracked beam' mode shapes. For validation purposes of their theoretical results, a 2D FEM was considered. A continuous beam vibration theory was developed with the help of Euler–Bernoulli's beam theory that can be applied to single-edged or double-edged open, cracked beams as proposed by Chondros et al. [12].

The cracked beam was accounted to be 1D continuum media. Hu Washizu Barr's variational formulation can be used to obtain a differential equation with associated boundary conditions. Continuous flexibility and displacement around the crack support the true design of the crack. It was estimated by using the fracture mechanics method. Two situations were considered: (1) A cracked beam which is made of aluminium with fatigue load initiated crack. (2) A cracked beam that is made of mild steel affected by a double-edged crack. They obtained the cracked beam's 1st natural frequency and validated the findings with the experimental data.

Ostachowicz and Krawczuk [13] emerged from the natural frequencies of a cracked cantilever beam bearing two cracks that remain open during vibration. They contemplated that single-edge defects are loaded to non-uniform loading, and cyclic loading is applied to the two-edge crack. Every finding is subjected to some assumptions which state that cracks remain open when they are externally loaded. Masoud et al. [14] considered a cracked beam that is prestressed and has both ends fixed. They investigated how crack depth affects the beam's lateral vibration. Their modal analysis data states that both crack depth and axial load cause significant coupling effects. This coupling effect is hampering the cracked beam's natural frequency of vibration. They also validated theoretical results with experimental data. A novel technique was derived by Shifrin and Ruotol [15], which can determine the beam's natural frequency(s) for finite open cracks chosen arbitrarily.

This new novel technique is capable of decreasing the calculation matrix's order. Hence, computation time can be reduced to a greater extent if compared to standard techniques. Standard techniques are used for the continuous beam model. Another novel approach was introduced by Kisa and Gurel [16]. Their technique was based on a numerical approach for analysing a cracked beam's free vibration, which had two types of cross sections: uniform c/s and stepped circular c/s, respectively. Two consecutive methods were used, like FEM and component mode synthesis method. They considered an assumption that separated components from crack's positions components are again attached due to the introduction of some flexibility matrices. These matrices are obtained from the mechanics of fracture.

Kessissoglou and Zheng [17] introduced a shape function that can fulfil the requirements of flexibility at the position of cracks. Their approach is more precise in finding beam's mode shapes due to the introduction of a shape function. They used FEM to determine the cracked beam's mode shapes and natural frequencies.

Fernnandez-Sa'ez et al. [18] emerged with one simpler approach based on scientist Rayleigh's method for applying Euler Bernoulli's beams with a single crack. Through this simpler approach, they evaluated both mode shape and closed form of fundamental frequency. They considered a polynomial function to take care of the crack's effect on the functioning of the cracked free beam.

Oyadiji and Zhong [19] enhanced the work done by Fernandez-Sa'ez et al. [18]. They considered an SS beam having a stationary mass, which is roving and has a single crack for calculating both mode shapes and natural frequencies. Lin and Chang [20] discovered a new method, which is the transfer matrix method. They considered a cantilever beam of a single crack to find both mode shapes and natural frequencies by using this new technique.

Khorram et al. [21] differentiated achievements of two damage identification techniques. Techniques are wavelet-based. Their ultimate ambition was to find both the size and location of an SS Beam acted upon by moving load. Khorram et al. [22] emerged one new method for detecting multiple number of cracks. They considered a SS Beam, which is acted upon by moving load along the longitudinal direction of the beam. They considered the Factorial design method and Continuous transform method together. They also considered that the beam would be deflected when a moving load is passed through the middle position of the beam.

Krawczuk and Ostachowicz [23] worked to find the relationship between amplitude of vibration, crack parameter and damage characteristics. Their findings show that cracks in beams are responsible for changes in the amplitude of vibration and changes in natural frequencies.

Using experimental methods, Chondros and Dimarogones [24] determined the cracked beam's natural vibration frequencies made of aluminium. Results obtained experimentally were verified with mathematical formulation, and the agreement was good.

Chati and Mukherjee [25] studied the beam's dynamic behaviours having cracks. It was concluded that there is a true difference between the dynamic behaviours of cracked and healthy beam structures.

Kam and Lee [26] took the help of FEM to identify both crack's depth & position after modelling and analysing a cracked cantilever beam. Chondros and Diamarones [27] studied the outcome of the dynamic properties of the single-edged cracked beam, which is supported by pin joints at the ends.

They also obtained mathematical expressions to identify how the cracked beam's natural frequency(s) is/ are affected by the crack. Rizoet et al. [28] determined a technique to identify crack depth and location by analysing a cracked cantilever beam's modes of vibration.

Matveenv and Bovsunovsky [29] obtained an expression of a cracked beam' lateral vibration by considering Euler-Bernoulli's beam theory. Their analysis was how modal natural frequency depends on the ratio between crack depth thickness and the ratio between crack position and length of the beam.

Radcliffe CP [30] worked to show how dynamic response characteristics are altered if any structure is subjected to damage. The presence of crack causes a reduction in stiffness as well as the natural frequency of lateral vibration. It also changes the damping property and mode shapes (32-33). Kausharet et al. [31] derived a method by which a cantilever beam' crack can be detected by using a frequency measurement technique.

D.R. Parhi and D.K. Agarwalla [32] analysed the modal parameters of a cantilever beam having crack(s) using both the Finite Element Method (FEM) and the experimental method. Dayal R Parhi and Irshad A Khan [33] considered two types of beam, fixed beam and cantilever beam, to study vibration characteristics when two cracks are present.

Dong Wei et al. [34] presented an analytical method to analyse the cracked beam's free vibration, which is made of Functionally Graded Material (FGM). The beam is subjected to not only axial loading and shear deformation but also rotary inertia. Celalettin Karaagac et al. [35] analysed how crack position and depth affect the fundamental frequencies of lateral vibration.

They also analysed lateral buckling of a single-edged cracked Euler-Bernoulli thin cantilever beam. Their investigation is based on both experimental method and numerical methods. For numerical analysis, they considered a finite element-based energy approach. Sadettin Orhan [36] numerically analysed cantilever beams with cracks subjected to forced and free vibration. They used a finite element-based numerical approach.

Samer Masoud Al-Said [37] suggested a simple algorithm that is supported by a mathematical model. By using this algorithm, they were able to detect the crack location and depth of the Euler–Bernoulli stepped cantilever beam subjected to a rigid disk at its end position. Their suggested identification algorithm focuses on shifting natural frequencies consisting of the first three due to the presence of a crack.

With this approach, location and depth can be estimated. In order to locate the crack, normalized frequency contours are plotted in terms of normalized crack location depth. Crack position depth is related when contours are intersected with planes of constant modal natural frequency.

G.M. Owolabi et al. [38] used an experimental method to analyse the crack's effect on vibration characteristics. To detect the crack's position, measured parameters were vibration amplitude and changes in the first three consecutive natural frequencies. A technique was suggested by S. P. Lele et al. [39] that can detect the position of the beam's crack for measurement of natural frequency. The proposed method only analyses frequency changes for the first mode.

The above literatures focuses on beams with either one or two surface cracks. Few papers focus on measuring vibration characteristics when surface crack(s) is/are changing with either position or thickness. A few papers also show how two surface cracks, varying in depth, alter the beam's vibration activities.

The variation in vibration phenomenon is dissimilar due to changes in crack opening size and position and crack depth variation. Beam stiffness value under lateral loading affects the natural frequency of vibration. Changes in crack opening size, crack position and depth are responsible for variations in beam stiffness value.

Many published researches on cantilever beam having crack(s) have not focussed how changes in three parameters like crack depth, position and opening size may affect natural frequency. Hence, there is still some research gap in the analysis of cracked beam vibration when three parameters change on a beam element.

In this research, the outcome of crack thickness, opening size, and position is analysed on the natural frequency of a cantilever beam with surface cracks up to several modes but restricted to five only. An Euler Bernoulli cantilever beam is considered. The natural frequency of vibration is determined for both crack-free and cracked beams.

3. Method

3.1. Theoretical Analysis of Cantilever Beam Vibration

A cantilever beam is supported at one end, and the other end is free of any support. As per Euler Bernoulli's thin beam theory, bending moment and transverse deflection are related by the following relationship:

$$M = EI\frac{d^2y}{dx^2}$$

For constant cross-section, the Equation of motion can be expressed as:

$$\frac{EI}{\rho A}\frac{d^4y}{dx^4} + \frac{d^2y}{dt^2} = 0 \quad (1)$$

$$c^2\frac{d^4y}{dx^4} + \frac{d^2y}{dt^2} = 0 \; \left(c = \sqrt{\frac{EI}{\rho A}}\right) \quad (2)$$

Equation (2) is a function of both time positions.

Hence it becomes: $y = w(x) * T(t)$ (3)

Above Eq. may be rewritten as follows:

$$\frac{c^2}{w(x)}\frac{d^4w(x)}{dx^4} = -\frac{1}{T(t)}\frac{d^2T(t)}{dt^2} \quad (4)$$

$$\frac{d^4w(x)}{dx^4} - \beta^4 w(x) = 0 \quad (5a)$$

$$\frac{d^2T(t)}{dt^2} + \omega_i{}^2T(t) = 0 \; \left(\beta^4 = \frac{\omega_i{}^2}{c^2} = \frac{\rho A\omega_i{}^2}{EI}\right) \quad (5b)$$

In order to obtain the solution of Ea.'s (5a-b), $w(x)$ This may be expressed as follows :

$$w(x) = C_1 * \cosh(\beta x) + C_2 * \sinh(\beta x) + C_3 * \cos(\beta x) + C_4 * \sin(\beta x) \quad (6)$$

After applying the boundary conditions, the following relationship is obtained :

$$\cosh(\beta L) * \cos(\beta L) + 1 = 0$$

The above expression has many solutions:

$$\omega_i = (\beta_i L)^2\sqrt{\frac{EI}{\rho A L^4}} \; (\omega_i : \text{Natural frequency in rad/sec}) \quad (7)$$

All material properties and dims. of beam with constant c/s are [39]:Young Modulus(E))=210GPa, Length (L)=.5m, Width(b)=.03m,Depth(h)=.004m, Area MOI (I)= 1.6×10^{-10} m^4, Mass per length (m)=.9336kg./metre, Density(ρ)= 7780 kg/m^3, Poisson's Ratio (ν)=.3.

Table 1 shows five natural frequencies as obtained from Equation (7).

Table 1. Modal frequency in hertz

Mode shape	Natural Frequency in Hz
1st	13.43
2nd	84.15
3rd	235.64
4th	461.78
5th	763.28

By using FEM soft. Workbench 17.0 numerical results of modal frequencies were obtained, as shown in Table 2.

Table 2. Modal frequency (Hertz)

Mode shape	Frequency (Hertz)
1st	13.177
2nd	83.554
3rd	234.12
4th	460.85
5th	762.57

The percentage of error between the above results is shown in Table 3.

Table 3. Error percentage

Mode shape	Th. freq. (Hz)	Num. freq.(Hz)	Error
1st	13.43	13.177	1.884%
2nd	84.15	83.554	0.708%
3rd	235.64	234.12	0.645%
4th	461.78	460.85	0.201%
5th	763.28	762.57	0.093%

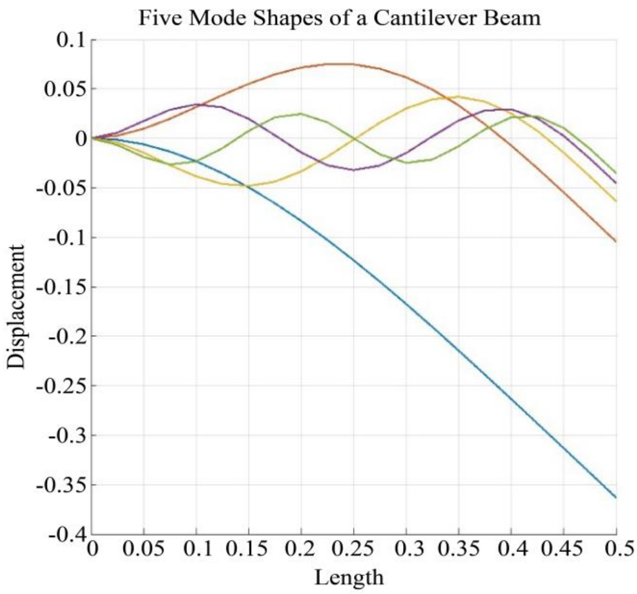

Fig. 1 Different modes of cracked free cantilever beam

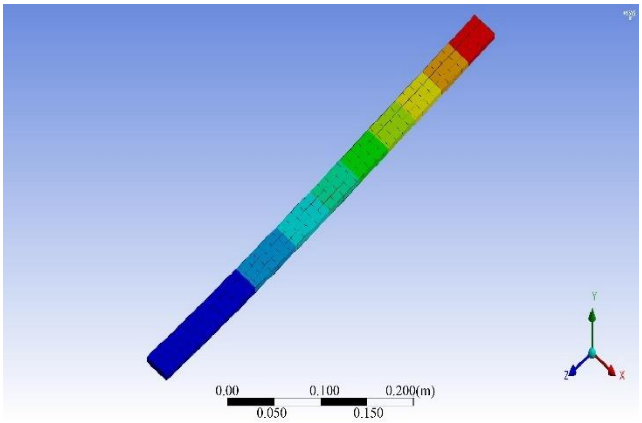

Fig. 2 First mode shape of a cracked free cantilever beam

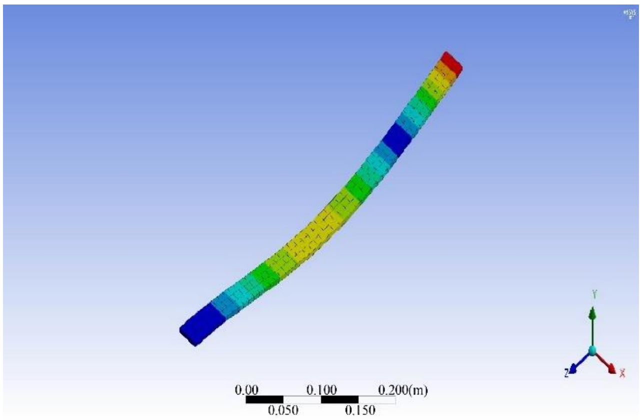

Fig. 3 Second mode's shape cracked free cantilever beam

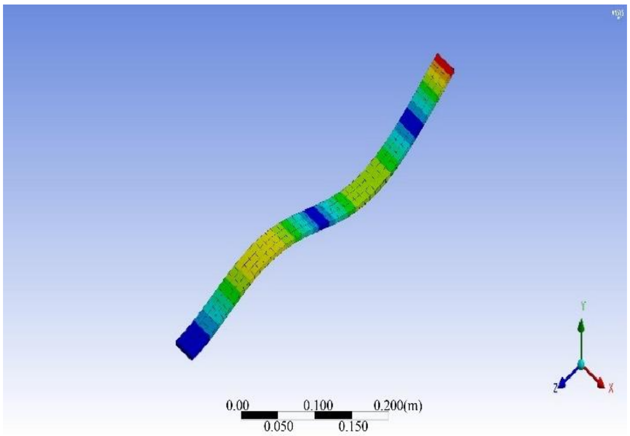

Fig. 4 Third mode's shape cracked free cantilever beam

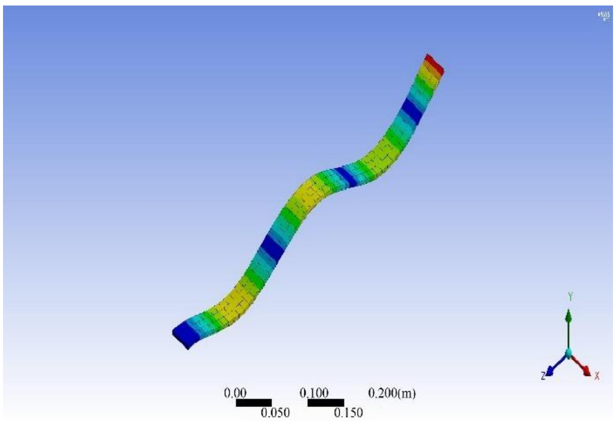

Fig. 5 Fourth mode's shape cracked free cantilever beam

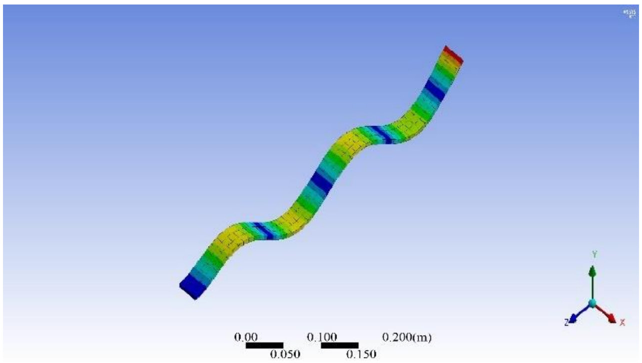

Fig. 6 Fifth mode's shape cracked free cantilever beam

3.2. Crack Model

The formation of a crack alters the geometrical properties of the beam. Due to this, the determination of the crack effect on structures like beams becomes a tricky task. Crack modelling is very important for simulation purposes. FEM is mainly chosen for the simulation of cracked beams to find modal frequencies. ANSYS Workbench 17.0 is used for simulation purposes, which works on the Finite Element Method; after modelling the cracked cantilever beam and considering both geometric and material linearity, modal simulation has been conducted. The crack is assumed to be a notch of V-shaped with an open on the top surface. When an alteration of frequency on different crack depths for different crack positions is formulated, the constant opening size of .5 mm on the beam's outer surface is considered. Variation of the natural frequency with different crack opening sizes is also analysed, keeping the crack depth at a constant value, i.e. $\frac{h}{2}$.

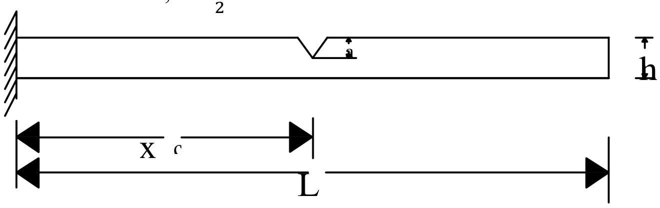

Fig. 7 Cracked cantilever beam having V-notch on the surface

4. Results and Discussions

Here, an analysis is done to determine how the modal natural frequencies of a cracked cantilever beam change with varying crack positions, crack depth and crack opening size, i.e., crack width.

Table 4. Variation of natural freq. (Hz) with depth of crack of 1st mode

Crack Location (Çc :-xc/L)	Crack Depth (H:- a/h)	Frequency (ωc /ω)
No crack		1.0
.2	.1	.999
	.3	.993
	.5	.974
.3	.1	.999
	.3	.995
	.5	.982
.5	.1	.999
	.3	.999
	.5	.994

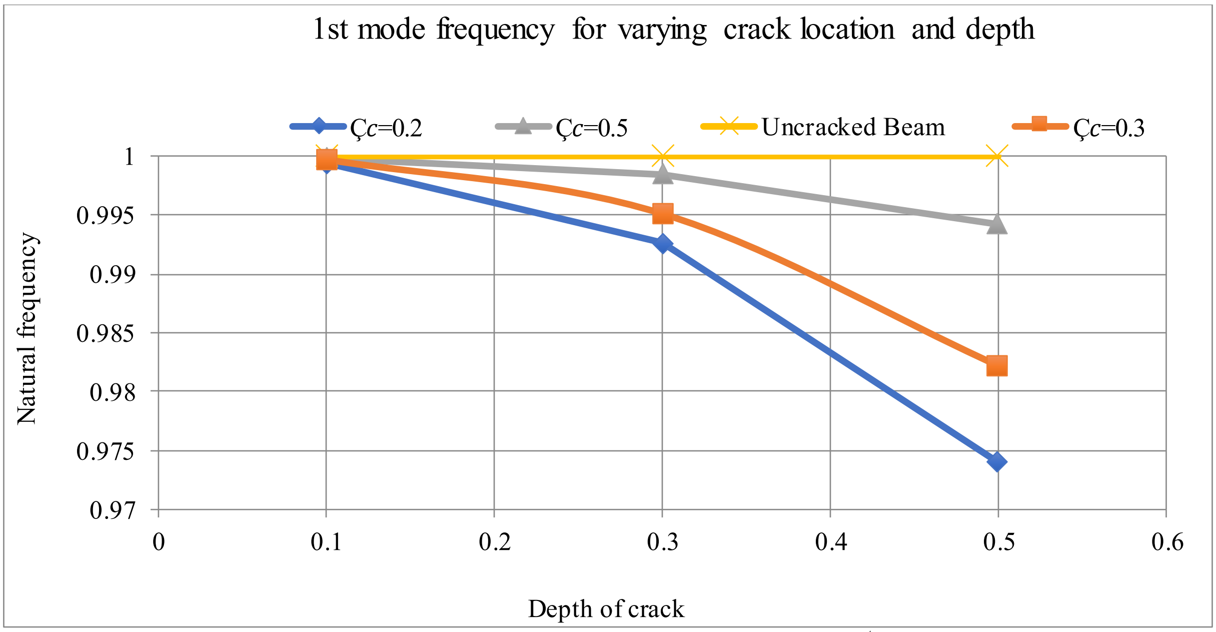

Fig. 8 Frequency and depth variation for cracked beam of 1st mode

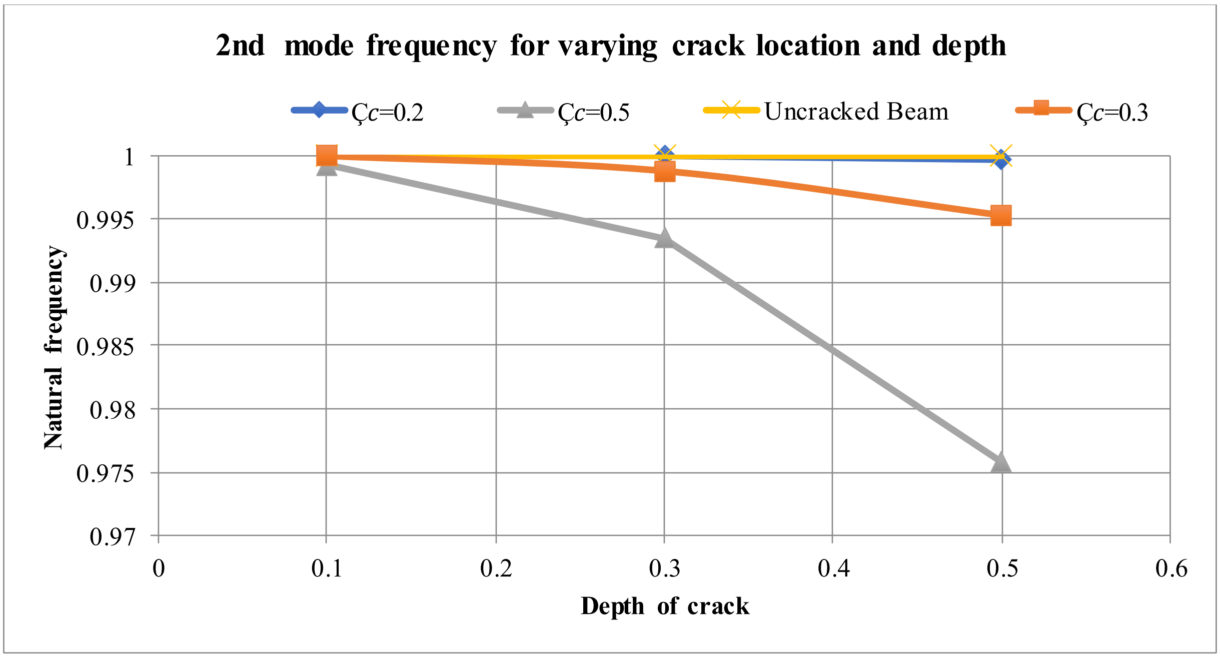

Fig. 9 Frequency and depth variation for a cracked beam of 2nd mode

Table 5. Variation of natural freq. (Hz) with depth of crack of 2nd mode

Crack Location (Çc :-xc/L)	Crack Depth (H:- a/h)	Frequency (ωc /ω)
No crack		1.0
.2	.1	1.0
	.3	1.0
	.5	.999
.3	.1	1.0
	.3	.999
	.5	.995
.5	.1	.999
	.3	.994
	.5	.976

The above Figure 8 clearly shows that when the crack is present at 100 millimetres from the fixed support of the beam, the reduction in natural frequency is highest. Drop in frequency is minimal when the crack is 250 millimetres from the support end. As per data obtained from Figure 1, vibration amplitude is highest at the free end, whereas zero is at the fixed end, but the bending moment is maximum.

From the above Figure 9 plot, the highest drop in frequency was observed at the middle location towards the longitudinal axis. The least reduction in freq. is observed at the crack's location of 100 millimetres from fixed support. The crack position of 150 millimetres from the support end also does not show a significant reduction in natural frequency.

Table 6. Variation of natural freq. (Hz) with depth of crack of 3rd mode

Crack Location (Çc :- xc/L)	Crack Depth (H:- a/h)	Frequency (ωc / ω)
No crack		1.0
.2	.1	.999
	.3	.998
	.5	.993
.3	.1	.999
	.3	.994
	.5	.979
.5	.1	.999
	.3	.999
	.5	.999

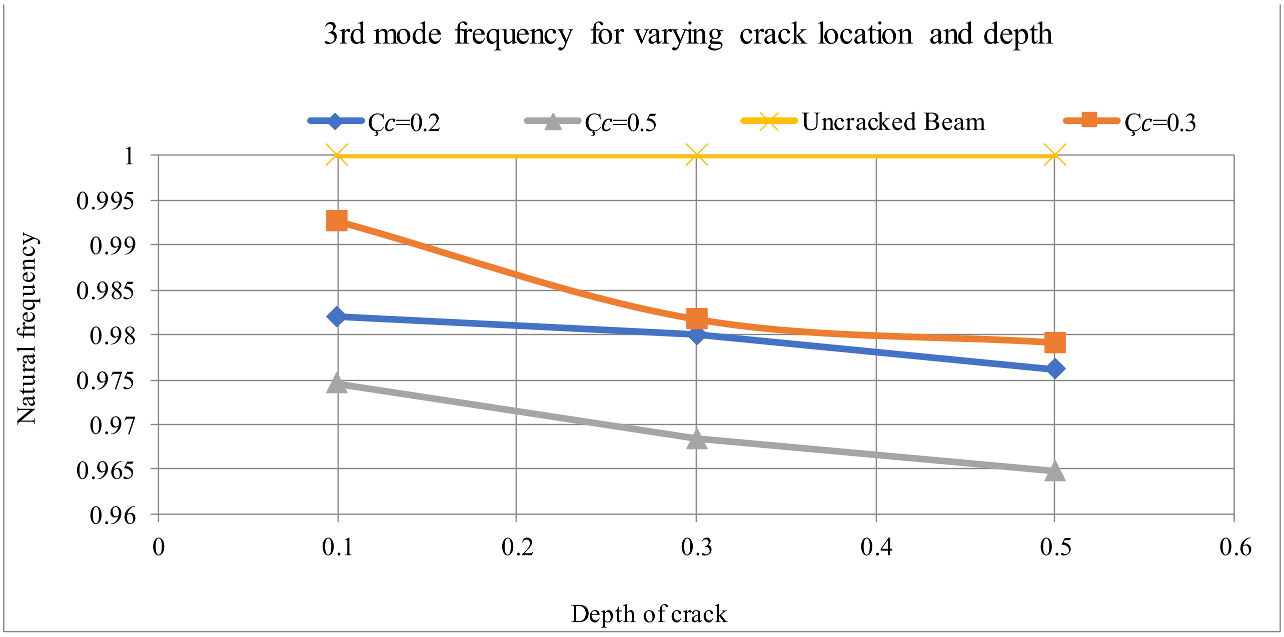

Fig. 10 Frequency and depth variation for cracked beam of 3rd mode

The above Figure 10 plot signifies that when the position of the crack is 150 millimetres from the support end, the drop in natural frequency reaches its highest value. The least reduction in frequency is observed at a crack position of 250 millimetres from the support end, i.e. at mid-location. The crack position of 100 millimetres from the support end also does not show a significant reduction in natural frequency as compared to the crack located at the middle of the length.

Table 7. Variation of natural freq. (Hz) with depth of crack of 4th mode

Crack Location (Çc :- xc/L)	Crack Depth (H:- a/h)	Frequency (ωc / ω)
No crack		1.0
.2	.1	.999
	.3	.994
	.5	.981
.3	.1	.999
	.3	.998
	.5	.992
.5	.1	.999
	.3	.994
	.5	.977

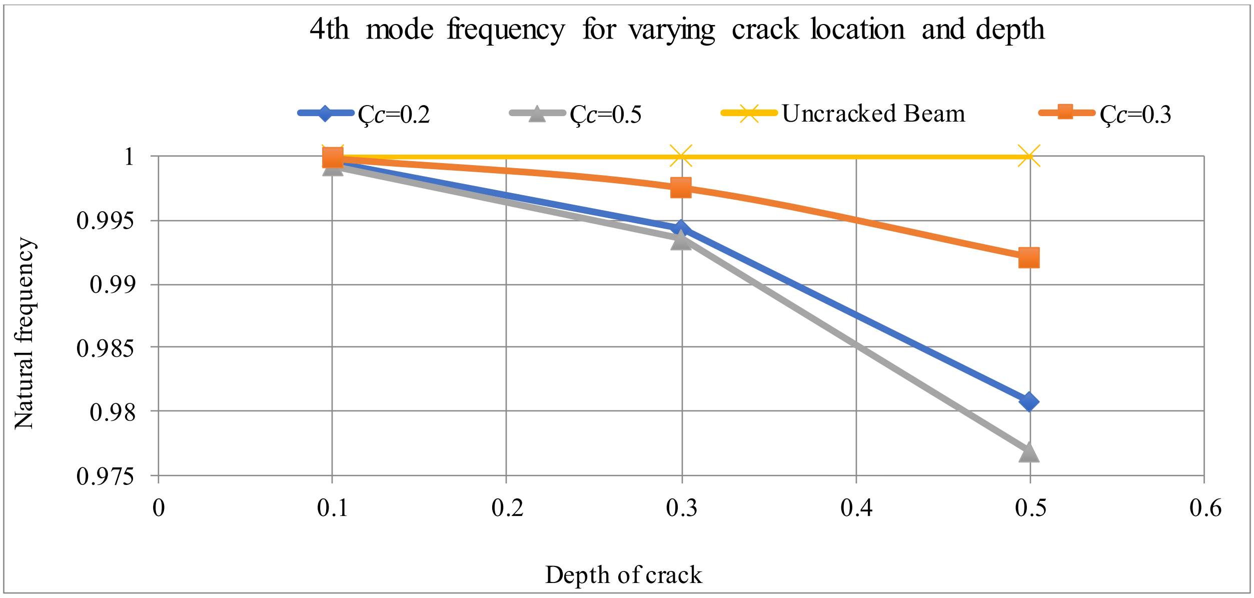

Fig. 11 Frequency and depth variation for a cracked beam of 4th mode

From the above Figure 11, it's observed that when the crack is positioned at mid location of the beam, the reduction in frequency is highest. For a crack location of 150 millimetres from the support end, a minimum frequency drop is observed. Due to the crack's presence at 100 millimetres from the support end, the drop in frequency is more than that of the crack position of 150 millimetres from support.

Table 8. Variation of natural freq. (Hz) with depth of crack of 5th mode

Crack Location (Çc :- xc/L)	Crack Depth (H:- a/h)	Frequency (ωc / ω)
No crack		1.0
.2	.1	.999
	.3	.994
	.5	.984
.3	.1	.999
	.3	.999
	.5	.998
.5	.1	.999
	.3	.999
	.5	.999

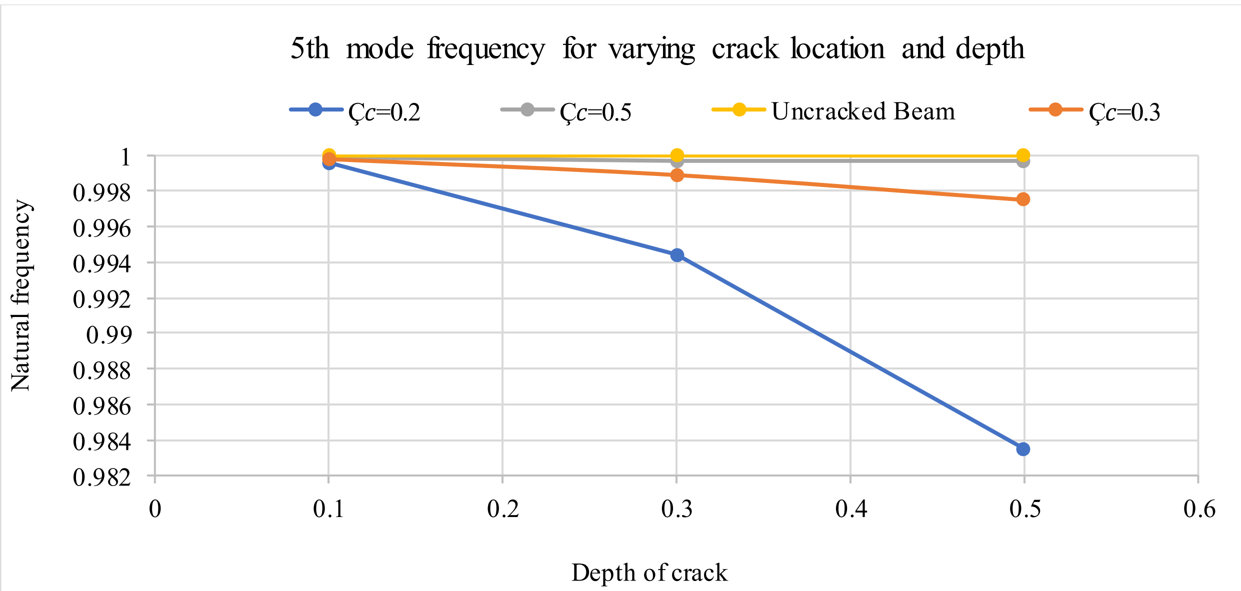

Fig. 12 Frequency and depth variation for cracked beam of 5th mode

The above Figure 12 plot signifies that when the crack is positioned at 100 millimetres from the support end, the natural frequency drop is highest. The lowest reduction in frequency is observed at a crack location of 250 millimetres from the fixed support end, i.e., the middle of the beam. The crack position of 100 millimetres from the support end also does not show a significant reduction in natural frequency like that of 100 millimetres from support as compared to the crack located at the middle of the length.

Table 9. Natural frequency variation with crack width for 1st mode

Crack Location ($Çc$: xc/L)	Crack Width (w/s)	Frequency (ωc / ω)
No crack		1.0
.2	.1	.9884
	.3	.981
	.5	.982
.3	.1	.991
	.3	.987
	.5	.987
.5	.1	.998
	.3	.995
	.5	.995

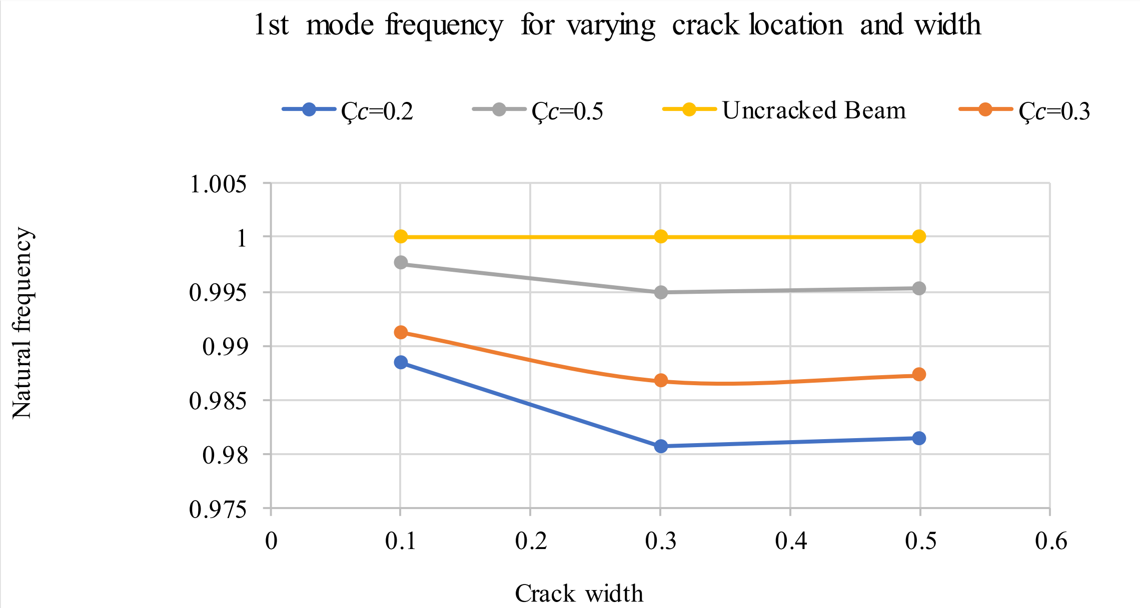

Fig. 13 Frequency and crack opening size variation for the first mode

Table 10. Natural frequency variation with crack width for 2nd mode

Crack Location ($Çc$=xc/L)	Crack Width (w/s)	Frequency (ωc / ω)
No crack		1.0
.2	.1	.999
	.3	.999
	.5	.998
.3	.1	.997
	.3	.996
	.5	.996
.5	.1	.989
	.3	.982
	.5	.983

From the above Figure 13 , it's observed that frequency reduction is maximum when the crack's presence is at 100 millimetres from the support end of the beam, and the crack's width becomes .15 mm. When the crack is present at 250 millimetres from the support end, the drop in frequency is minimal with a change in crack width. During a change in crack opening size from 0.15 mm to 0.25 mm, variation of natural frequency is negligible.

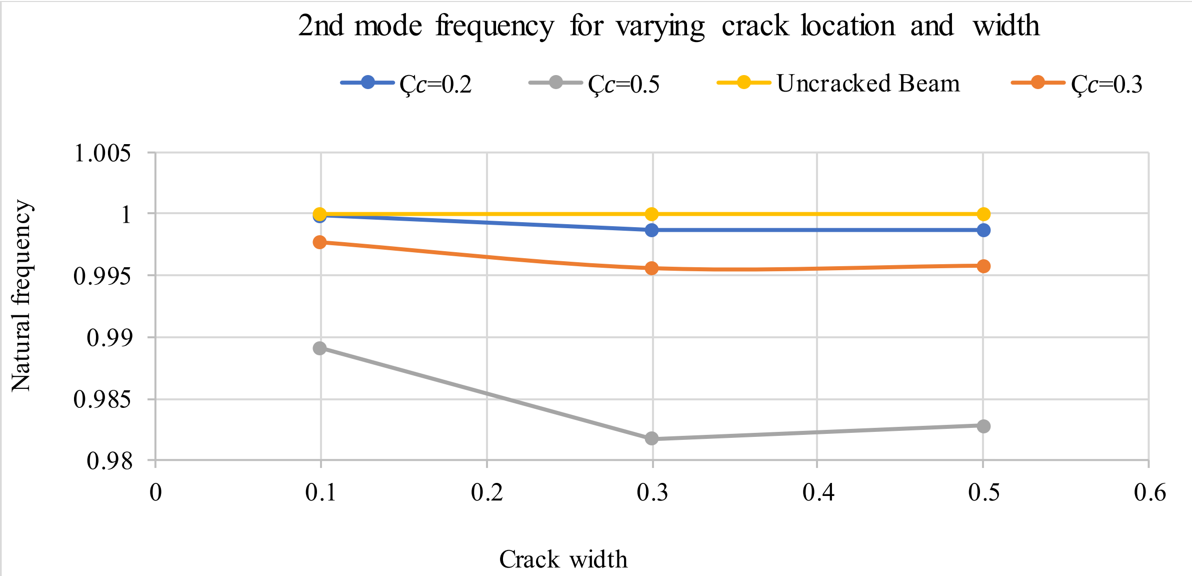

Fig. 14 Frequency and crack opening size variation for second mode

From the above Figure 14 plot, the highest drop in frequency is found at the midpoint of the beam when the crack width reaches a value of 0.15 mm. The least freq. reduction is observed at a crack location of 100 millimetres from the support end. In this mode shape, during a change in crack opening size from 0.15 mm to 0.25 mm, variation of natural frequency is also negligible.

Table 11. Natural frequency variation with crack width for 3rd mode

Crack Location (Çc=xc/L)	Crack Width (w/s)	Frequency (ωc / ω)
No crack		1.0
.2	.1	.997
	.3	.994
	.5	0.994
.3	.1	0.991
	.3	0.985
	.5	0.985
.5	.1	0.997
	.3	0.998
	.5	0.999

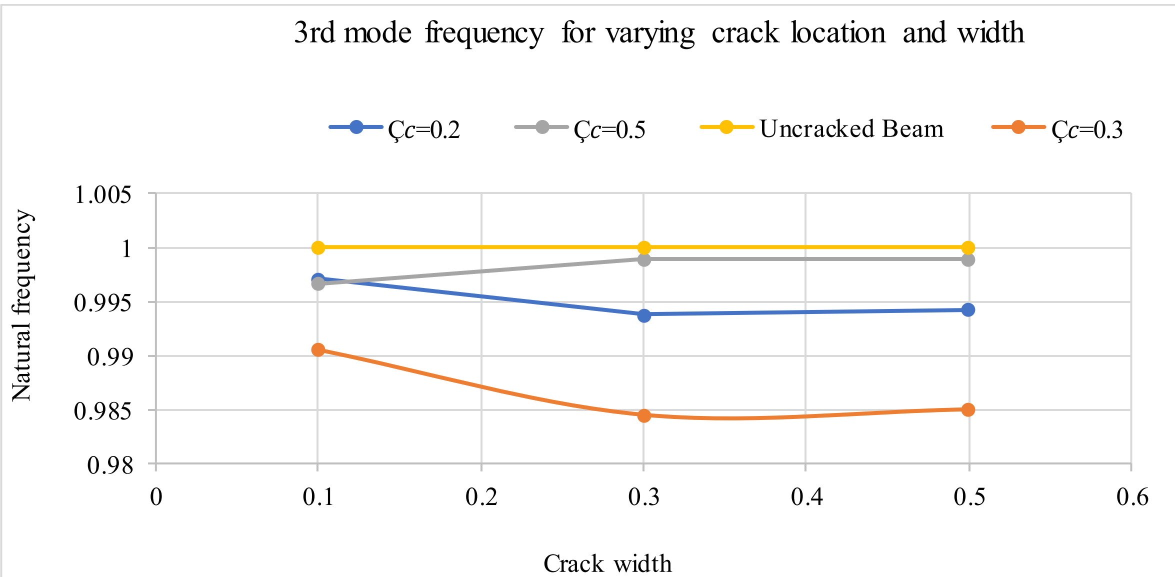

Fig. 15 Frequency and crack opening size variation for the third mode

The above Figure 15 plot clearly signifies that the highest drop in natural frequency is observed when the crack is 150 millimetres from the support end, and the crack's width is 0.15 mm. The least reduction in frequency is observed at a crack location of 250 millimetres from the support end, i.e., the middle position of the beam. For this mode, during a change in crack opening size from 0.15 mm to 0.25 mm, the variation of natural frequency is much less and can be neglected.

Table 12. Natural frequency variation with crack width of 4th mode

Crack Location (Çc=xc/L)	Crack Width (w/s)	Frequency (ωc/ω)
No crack		1.0
.2	.1	.992
	.3	.985
	.5	.986
.3	.1	.997
	.3	.994
	.5	.994
.5	.1	.991
	.3	.983
	.5	.984

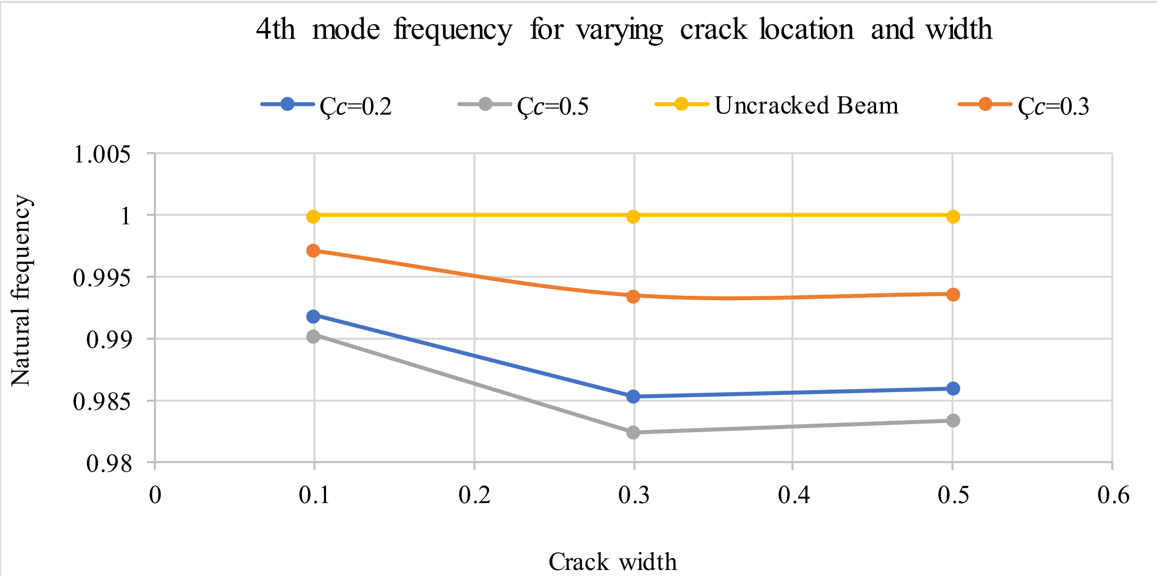

Fig. 16 Frequency and crack opening size variation for fourth mode

The above Figure 16 clearly signifies that when the crack is positioned in the middle of the beam, the crack's width is .15 mm, and frequency reduction reaches the highest value. The frequency drop is minimal when the crack is located 150 millimetres from fixed support. When the crack position is at 100 millimetres from fixed support, the frequency drop is less than that of the crack position of 250 millimetres from the support end. During a change in crack opening size from 0.15 mm to 0.25 mm, the variation of natural frequency is quite less.

Table 13. Natural frequency variation with crack width of 5th mode

Crack Location (Çc=xc/L)	Crack Width (w/s)	Frequency (ωc / ω)
No crack		1.0
.2	.1	.994
	.3	.987
	.5	.988
.3	.1	.999
	.3	.997
	.5	.997
.5	.1	.998
	.3	.998
	.5	.997

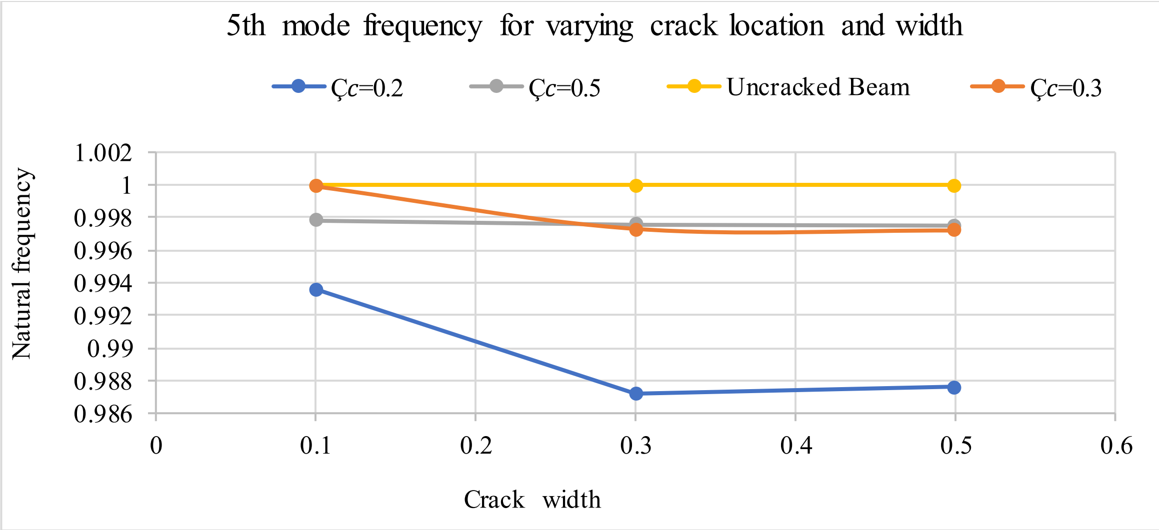

Fig. 17 Frequency and crack opening size variation for the fifth mode

The above Figure 17 plot clearly signifies that the highest drop in natural frequency occurs 100 millimetres from the support end when the width is 0.15 mm. The least reduction in frequency is observed at the crack location of 250 millimetres from the fixed support end and 150 millimetres from the support end. In this mode shape, during a change in crack opening size from 0.15 mm to 0.25 mm, variation of natural frequency is quite negligible.

5. Conclusion

Based on the Finite Element Analysis, the present study focuses on the conclusions below.

- ❖ Significant changes in natural frequency occur based on crack depth, position, and opening size.
- ❖ When the crack's locations are fixed, the beam's vibration frequencies are not directly proportional to the crack's depth.
- ❖ It is found that changes in freq.(s) are/ are related to crack opening size, depth, location and mode shape.
- ❖ At the support end of the beam, the drop in frequencies is maximum. It is proved that the fall of natural frequencies will be highest where the bending moment value is maximum.
- ❖ The cracked cantilever beam's natural frequencies reduce with the rise in crack opening size up to a certain limit when both crack position and crack depth are fixed. After that, variation is very minimal and can be neglected.
- ❖ When the depth of the crack is constant, but crack locations from the free end are varied, natural frequencies also change.
- ❖ At a particular crack's depth, changes in natural vibration frequency are reduced when the crack's location moves from the support end.
- ❖ Crack's effect on modal frequencies is different for all mode shapes.
- ❖ The amplitude of transverse vibration rises due to the rise in the crack's depth.
- ❖ The cracked beam's natural frequencies increase when a position shifts towards the free end for the constant value of crack depth.
- ❖ With shifting of crack location, vibration amplitude with respect to higher freq.(s) rises, whereas at lower freq. (s) reduces.

Terminology

M	BM acted on the beam
L	Beam's length
ν	Poisson's ratio
h	Height (beam)
b	Width (beam)
t	time (sec.)
y	Beam lateral deflection
ρ	Mass density.
E	Young's modulus
x	Longitudinal direction
I	Area MOI
$\zeta_c = \frac{x_c}{L}$	Crack's location (non-dimensional)
x_c	Crack's location (dimensional parameter)
H	Crack's depth (non-dimensional)
a	Crack's depth along the thickness
m	Mass per length
ω_i	Modal frequency
A	Beam's area of c/s
w	Crack opening size
ω_c	Cracked beam frequency
s	Original crack width

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